He finds that

$$Z_{oe} - Z_{oo} = \frac{120}{\sqrt{\epsilon_r}} ln \coth \frac{\pi s}{2b} \quad (24)$$

$$Z_{oe} + Z_{oo} = \frac{120}{\sqrt{\epsilon_r}} ln \coth \frac{\pi d}{4b} \quad (25)$$

where s = center to center spacing of the wires, d = wire diameter, b = ground plane spacing, and ϵ_r = relative dielectric constant of medium surrounding the wires. For configurations of coupled lines that are not amenable to analysis it is always possible to determine Z_{oo} and Z_{oe} from measurements of the static capacity of the strips. Thus

$$Z_{oo} = \frac{100\sqrt{\epsilon_r}}{3C_{oo}} = \frac{100\sqrt{\epsilon_r}}{3[C_{22} - C_{23}]} \quad (26)$$

while

$$Z_{oe} = \frac{100\sqrt{\epsilon_r}}{3C_{oe}} = \frac{100\sqrt{\epsilon_r}}{3[C_{22} + C_{23}]} \quad (27)$$

where

C_{oo} = one half the capacity between the two center conductors ($\mu\mu$f/cm),
C_{oe} = one half the capacity between the parallel combination of the two center conductors and ground ($\mu\mu$f/cm),
C_{22} = coefficient self capacity ($\mu\mu$f/cm) of either of the center conductors,
C_{23} = coefficient of induction ($\mu\mu$f/cm) between the two center conductors (a negative quantity).

The capacity C_{22} is easily measured as the capacity between either strip and ground when the other strip is grounded, while the capacity $C_{22} + C_{23}$ is just one half the capacity between the parallel combination of the two center conductors and ground.

ACKNOWLEDGEMENT

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Absolute Measurement of Receiver Noise Figures at UHF*

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Summary—**Absolute measurements of noise-figures in the UHF range are described, using hot and cold thermal sources as standards. It was found that the noise temperature of the *T*-5 6 watt fluorescent tube is 16.1 ±0.6 db above *kT*Δν. Noise diodes were found to be in error at these frequencies by approximately 1 db.**

INTRODUCTION

THE noise figure of a receiver is conventionally defined as

$$F = \frac{\text{Noise power output of receiver}}{\text{Receiver gain} \times \text{available noise power from source}}.$$

It is measured either by comparing the noise power produced by the receiver with a signal from a calibrated signal generator, or with an external source of noise power of known intensity. The signal generator technique is straightforward but difficult to do precisely and requires that the "noise bandwidth" of the receiver be determined.[1] The second technique, which uses a calibrated noise generator, is simpler to apply and is usually capable of greater precision provided that a reliable standard of noise is available. Two types of noise generators which are primary standards, and hence self-calibrating, are the thermal source and the noise diode. The thermal source is simply a resistor at some temperature, T, which is capable of delivering noise power $kT\Delta\nu$ to some external device. Although simple in principle it is obviously restricted to laboratory use and has relatively low noise output. The diode noise source is a temperature limited diode. It can be shown[2] that the diode current contains a shot noise component whose means square value is

$$\bar{i}^2 = 2eI\Delta\nu$$

in the frequency interval $\Delta\nu$. e is the electronic charge and I the dc diode current. If the diode is shunted by a resistance R the combination is a noise generator with available power $2eIR\Delta\nu$. At frequencies high enough, so that the transit time of the electrons may not be neg-

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[1] G. E. Valley and H. Wallman, "Vacuum Tube Amplifiers," McGraw-Hill Book Co., New York, p. 695; 1948.

[2] *Ibid.*, p. 701.

lected, a suitable reduction factor $\psi(\nu)$ must be applied. Johnson[3] has calculated this correction factor for the coaxial type of noise diode subject to some simplifying assumptions, and this correction is generally applied when using this type of noise diode in the UHF spectrum. Commercial diode noise generators for this frequency range make use of the Bendix TT-1 coaxial noise diode shunted by a lumped resistor equal to the line impedance. Such devices are relatively broadband in impedance match. However, the fractional error in available noise power output will be equal to the voltage standing wave ratio, so that a mismatch of 1.25 is equivalent to 1 db uncertainty in output. Careful adjustment of the terminating impedance should reduce or eliminate this error.

Gas discharge devices have also been used as a convenient source of noise at microwave frequencies.[4,5] In its usual form this device consists of a gas discharge tube, usually a commercial fluorescent lamp, electrically matched into a waveguide. The discharge acts substantially like a black body radiator whose temperature is the electron temperature of the discharge. Approximate agreement was reported by Easely and Mumford between electron temperatures found from probe measurements and the equivalent temperatures deduced from noise measurements.[6] (The method of calibrating the noise measuring equipment was not stated but presumably a signal generator was used.) However, at the present time the gas discharge tube must be regarded as a secondary reference whose noise output is to be determined by direct comparison with a primary noise standard.

We have used diode noise generators for some time in measuring the noise figure of UHF receivers in the vicinity of 400 mcps. The terminating impedances were trimmed up so that errors due to impedance mismatch were negligible at the operating frequency. Nevertheless we observed a discrepancy between these measurements and those made with a fluorescent tube source (using the previously accepted figure[6] of 15.84 db excess noise at 40°C). The noise figures measured with the diode noise source seemed to be one or two db higher than those observed with the gas discharge source. Accordingly it was decided to build some thermal noise sources for independently checking these observations.

To measure the noise figure with a thermal source one first observes the noise power output of the receiver when fed from a source impedance at room temperature, T_0, and again when the source impedance is at some other temperature T. If the noise power outputs in these two cases are N_0 and N respectively, then the noise figure is given by

$$F_{T_0} = \frac{\dfrac{T}{T_0} - 1}{\dfrac{N}{N_0} - 1}.$$

It is customary to refer all measurements to a room temperature of 290°K so that F becomes:

$$F = \frac{\dfrac{T}{290} - 1}{\dfrac{N}{N_{290}} - 1}.$$

If the measurements are made at an ambient temperature, T_0, other than room temperature the noise figure is given by

$$F = \frac{\dfrac{T}{290} - 1 - \dfrac{N}{N_0}\left[\dfrac{T_0}{290} - 1\right]}{\dfrac{N}{N_0} - 1},$$

where N_0 is the noise power output when the source impedance is at temperature T_0.

In order to obtain reasonable precision N/N_0 should not be too close to 1. If we assume typical noise figures of the order of 5 db (3.2 arithmetically) and $T\sim$ 1,000°K, N/N is approximately 1.6 and F can be determined to within 0.1 db. Temperatures of the order of 1,000°K are convenient to work with so this technique is seen to be entirely practical for reasonably low noise-figures.

Instead of making T large compared to T_0, we can make it small. For the limiting case of $T=0$, and $F=3.2$, $N/N_0=0.69$. F can be determined to within about 0.25 db. Temperatures of the order of 4°K may be obtained with liquid helium and are low enough for this purpose. In our work both high temperature and low temperature thermal sources were employed and reasonable agreement was observed between measurements.

High Temperature Source

The high temperature source used was a long lossy helical line constructed by winding a helix of 42 turns of no. 27 Karma wire on a ceramic form $1\frac{1}{2}$ inches in diameter. Details of construction are shown in Fig. 1. The form was constructed of commercial "Lava"[7] which was first turned down to size and threaded and subsequently fired. The helix pitch is 20 turns per inch. The ceramic form is encased in a snugly fitting steel tube, the spacing between wire and shield being 0.016 inch. The helix is directly coupled to a stainless steel 50 ohm line. It was designed for a nominal impedance of 50 ohms and the input voltage standing wave ratio, when

[3] H. Johnson, "A Co-axial line noise diode source for UHF," *RCA Rev.*, vol. 8, p. 169; March, 1947.
[4] W. W. Mumford, "A broad band microwave noise source," *B. S. T. J.*, vol. 28, p. 608; October, 1949.
[5] H. Johnson and K. R. DeRemer, "Gaseous discharge super-high frequency noise source," Proc. IRE, vol. 39, p. 908; August, 1951.
[6] M. A. Easley and W. W. Mumford, "Electron temperature vs noise temperature in low pressure mercury-argon discharges," *J. Appl. Phys.*, vol. 22, p. 846; September, 1951.
[7] This is a talc-like material sold by the American Lava Company.

hot, is 1.5. An electric furnace was built around the helical line after assembly. In practice the source is continuously maintained at a temperature of about 1,000°K. The temperature is monitored by three thermocouples situated along the length of the furnace. The temperature differential along the length of the furnace is less than 9°.

Since the noise-figure of a receiver depends on the effective source impedance this must be the same for the thermal and the room temperature sources. The admittance of the thermal source was measured with a General Radio Type 1602B Admittance Meter. Thermal sources were equipped with *GR* type 874 connectors so the admittance meter could be connected directly to the source output terminals and the admittance, referred to those terminals, measured directly without the use of any connecting cable. The room temperature source is then used together with an impedance transformer, also equipped with *GR*-874 connectors, and the transformer adjusted so that the impedance of the room temperature source is identically equal to that of the thermal source. The physical arrangement is shown in Fig. 2. The noise figure of the receiver is then determined for the case where the source impedance is equal to that of the thermal source. For other values of source impedance an appropriate impedance transformer could have been inserted between the thermal source and receiver. In practice, however, the thermal sources are not used for routine noise-figure measurements but rather as standards for calibrating noise-diodes and fluorescent tube sources. For this purpose it is more convenient to transform all other impedances to be identical with that of the thermal source. After the equivalent noise temperature of the secondary source has been determined it is, of course, used without any impedance transforming device. Any attenuation in the impedance transformer must be allowed for. In our case this correction was 0.1 db.

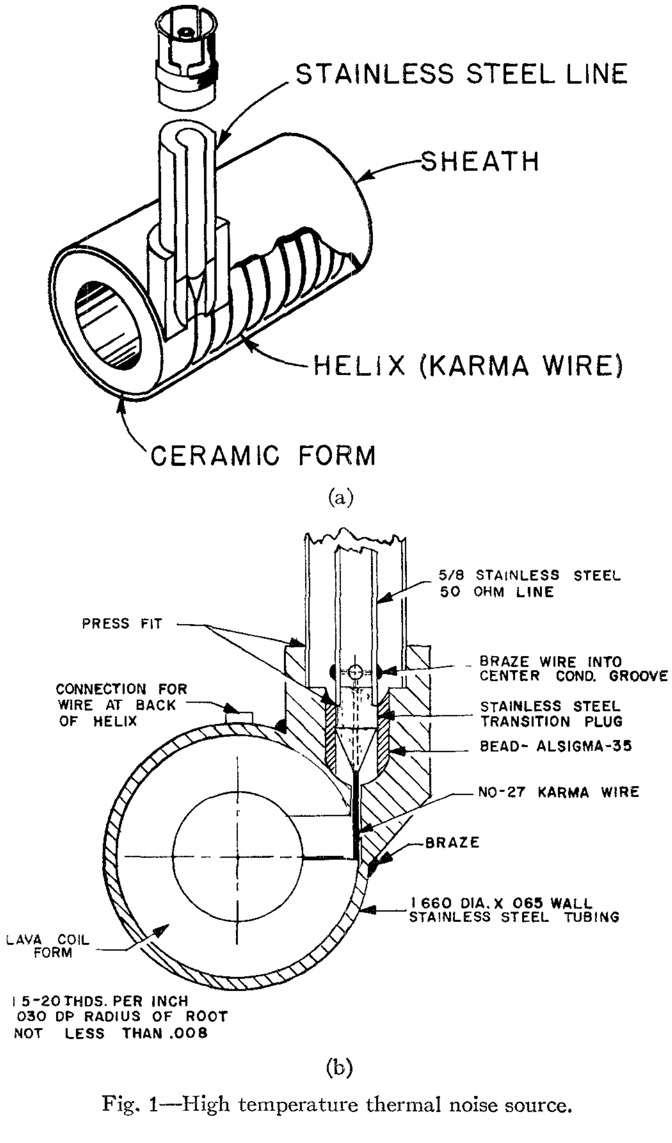

Fig. 1—High temperature thermal noise source.

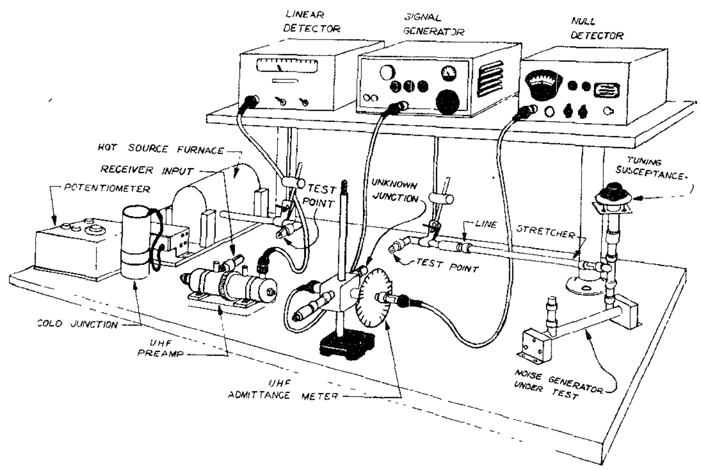

Fig. 2—Physical arrangement of equipment for measuring noise temperatures with thermal source.

Low Temperature Source

The low temperature source is illustrated in Fig. 3. It consists simply of a small 50 ohm plantinized glass resistor which terminates a 50 ohm transmission line. The resistor is immersed in liquid helium so that its temperature is therefore 4.2°K. The transmission line is made of stainless steel, chosen for low heat conductivity, which is silver plated to reduce the electrical attenuation. In using this device a reasonable amount of care must be used to cool it down slowly and to allow it to warm up slowly after use, otherwise the thermal shock may break the fragile resistor. A platinized glass resistor which is 50 ohms at room temperature falls to about 35 ohms at 4.2°K. (The small temperature coefficient is due to the fact that most of the resistance is caused by boundary scattering.) This source is much simpler to construct than the hot source but of course requires a supply of liquid helium.

Stem Corrections

Since the thermal sources must be connected to the receiver by transmission lines along which the temperature varies continuously from source to receiver.

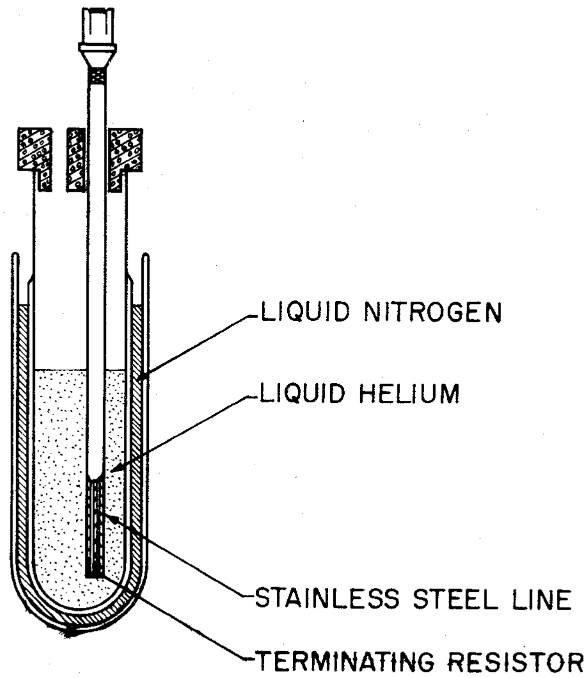

Fig. 3—Low temperature source.

it is important to know the magnitude of the appropriate stem correction. If we have a noise source, at temperature T, which is connected to a receiver through a four terminal network which is at temperature T_2, and has a gain G, the power delivered is given by[8]

$$P = GkT\Delta\nu + (1 - G)\,kT_2\Delta\nu.$$

If the network is a transmission line whose gain (attenuation) and temperature are both a function of position, then it may be shown that the noise power delivered is

$$kT\Delta\nu \exp\left(-\int_0^l \alpha(x)dx\right) + k\Delta\nu \int_0^l T(x)\alpha(x) \exp\left(-\int_x^l \alpha(\xi)d\xi,\right) dx$$

where (x) the attenuation constant at point x, $T(x)$ the temperature at point x, and l = the total length of the line. The effect of the correction is to lower the apparent temperature of the hot source and raise the apparent temperature of the cold source. If $T(x)$ and $\alpha(x)$ are given as empirical data the correction may be calculated by numerical integration. However, it is sometimes more useful to calculate outside limits to the correction which may be done somewhat more simply. If for the case of the hot source, it is assumed that the connecting line is all at room temperature but that the attenuation of the line is that corresponding to the high temperature, then the corresponding correction will certainly be larger than the true correction. This upper limit was calculated to be 9° for a source temperature of 965°K and hence the correction is less than 0.05 db. In the case of the cold source the corresponding upper limit to the correction is of the order of 0.025 db and therefore also negligible.

Results of Measurements with Thermal Sources

Although the thermal noise sources may be used directly to measure the noise figure of a receiver, it is usually more convenient to use them as primary standards for calibrating working standards. The hot source is preferable for day-to-day work. The chief utility of the cold source is to provide a cross-check.

We have used two types of working standards. These are the Bendix TT-1 diode noise generator (in a modified commercial instrument) and the fluorescent-tube gas-discharge noise generator. The diode source was compared with both hot and cold sources at a frequency of 425 mcps. For noise figures of the order of 5 db it was found that measurements made with the diode noise generator were on the average 1.14 db higher than with the hot source and 1.28 db higher than with the cold source. The transit time correction calculated by Johnson[3] is only 0.3 db at this frequency. It must be concluded that this correction is too small by 0.7–1.0 db or that there is some other source of error inherent in the diode. In our work with the diode generator we have applied a correction of −1.2 db at this frequency.

The fluorescent tube sources use a standard 6 watt, $T5$, cool white fluorescent lamp. Coupling to the discharge was effected by a helical line wound directly on the glass envelope of the tube. Some details of the design are given in Fig. 4. The design used was evolved from an earlier version due to Hill Montague of the Naval Research Laboratory.[9] The helix dimensions were chosen to match into a 50 ohm line. Pertinent data on the input impedance and attenuation through the discharge areas follows.

Input Admittance*

Frequency	Hot	Cold	Attenuation
200	21.7—j 1.0	22.6—j 2.2	13 db
400	20.6—j 0.7	25.0—j 3.75	26 db
600	20.7—j 0.8	36 —j 1.6	32 db
900	22.9—j 7.8	16.5—j 9.0	50 db

* Referred to input terminals in 20 millimho line.

The hot admittance is that seen at the input terminals when the helical line is tightly coupled to the gas discharge. The cold impedance is that of the termina-

[8] A. van der Ziel, "Noise," Prentice-Hall, Inc., New York, p. 16; 1954.

[9] Since this paper was written a description of his work has appeared in NRL Report 4560.

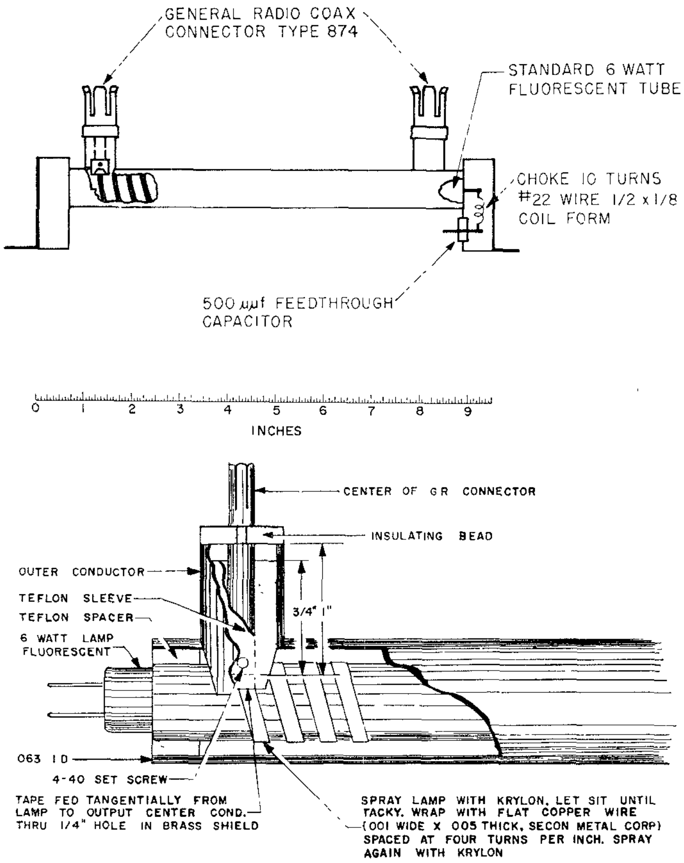

Fig. 4—Fluorescent tube noise source.

tion transformed back to the input terminals when the discharge is turned off. The helical line parameters were adjusted to secure a near optimum match into the line when loaded with the discharge. The characteristic impedance of the line is complex under this condition and consequently the cold impedance appears somewhat mismatched. For very precise measurements the room temperature measurement should be made using a separate matched load and not the cold impedance of the fluorescent tube source.

The fluorescent lamp was operated with a discharge current of 100 ma. DC and the output end was always in the positive column end of the discharge. The average excess noise output for 9 tubes was 16.1 db above $kT\Delta\nu$ with a spread of 15.85 to 16.31 db among the group. This is for a bulb temperature of 40°C. Mumford[4,6] has given the temperature coefficient of the excess noise for these tubes.

In some tubes there was a decided asymmetry of the following sort. If the polarity of the discharge were reversed and the same time output and termination ends switched so that the output end was always adjacent to the anode then variations in noise output of as much as 0.4 db were observed in some tubes. It was suspected that this might have been due to structural asymmetries in the lamp construction. Several special sources were therefore constructed using longer tubes, both 8 watt and 13 watt types, while keeping the helix length the same. This kept the ends of the discharge well out of the field of the helix. The average excess noise for five tubes was 16.4 db or 0.3 db higher than for the 6 watt tubes. For each of these tubes measurements were made using all four permutations of discharge polarity and output position. The maximum spreads in excess noise for the five tubes were 0.4 db, 0.3 db, 0.2 db, 0.1 db, and 0.1 db. Since the maximum spread was the same as that observed with the shorter tubes, these occasional departures from the mean cannot be definitely assigned to structural asymmetry. Furthermore the difference in noise output of the 6 watt and the 8 and 13 watt tubes emphasizes the fact that the commercial fluorescent tube, although cheap and convenient, is not an invariable noise standard. Discharge tubes using pure argon, neon, or helium are undoubtedly to be preferred except for cost.

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Note

Further measurements of the noise output of standard 6-watt fluorescent lamps have shown that the apparent excess noise may vary by as much as −0.6 db over the course of a day. This effect seems to be correlated with fluctuations in the mercury vapor pressure but does not depend in any consistent fashion upon the bulb temperature. Some special lamps were obtained from Sylvania in which no mercury had been deliberately added. These provided a noise output of 16.3 db constant to within about 0.1 db or less. These tubes contain a small residual mercury vapor pressure which is cleaned up by the fluorescent coating after 20 to 30 hours of operation. After clean-up the tubes require a higher starting voltage.